Color-Shift-Keying Constellation-Design Case Studies

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***Abstract*— Color shift keying (CSK) is an emergent modulation for visible light wireless communication, whereby the light color is visibly constant. This paper presents several case studies of color-shift-keying constellations designed using various methods of constrained optimization to maximize the minimum Euclidean distance among the constellation symbols.**

I. Mathematical Statement of the Color-Shift-Keying Constellation Design as an Optimization Problem [1]

The current problem is to design an M-ary constellation, with constellation points $\{\mathbf{s}_m, \forall m = 1, 2, \cdots, M\}$, such that the constellation would give the largest "minimum distance" after passing through a channel with a known channel matrix of $\mathbf{H}$, while having each constellation symbol subject to a set of transmission power constraints and subject to one common color-constraint. The natural number M is generally a known positive power of 2 and is at least 4 (i.e., M could be preset to a known value of 4 or 8 or 16, etc.).

Each constellation symbol $\mathbf{s}_m$ is a 3×1 vector,

$$\mathbf{s}_m := \begin{bmatrix} [\mathbf{s}_m]_\mathrm{r} \\ [\mathbf{s}_m]_\mathrm{g} \\ [\mathbf{s}_m]_\mathrm{b} \end{bmatrix}, \tag{1}$$

with elements of non-negative scalars whose values are to be designed by maximizing the constellation's "minimum distance".

This maximization may be mathematically stated as

$$\arg \max_{\{\mathbf{s}_m, \forall m\}} \underbrace{\min_{\forall \mathbf{s}_j \neq \mathbf{s}_k} \overbrace{\|\mathbf{H}(\mathbf{s}_j - \mathbf{s}_k)\|_2}^{d_{j,k} :=}}_{\text{minimum distance}}, \tag{2}$$

where

$$\mathbf{H} := \begin{bmatrix} h_{\mathrm{r,r}} & h_{\mathrm{r,g}} & h_{\mathrm{r,b}} \\ h_{\mathrm{g,r}} & h_{\mathrm{g,g}} & h_{\mathrm{g,b}} \\ h_{\mathrm{b,r}} & h_{\mathrm{b,g}} & h_{\mathrm{b,b}} \end{bmatrix} \tag{3}$$

represents a 3×3 matrix of real-value scalars that are prior known.

The above maximization is subject to all six following constraints of (4) to (8):

Each constellation symbol $\mathbf{s}_m$ must satisfy the following "power constraints":

$$[\mathbf{s}_m]_\mathrm{r} \in [0, I_\mathrm{r}], \tag{4}$$
$$[\mathbf{s}_m]_\mathrm{g} \in [0, I_\mathrm{g}], \tag{5}$$
$$[\mathbf{s}_m]_\mathrm{b} \in [0, I_\mathrm{b}], \tag{6}$$
$$\|\mathbf{s}_m\|_1 \in [L_{\min}, L_{\max}], \tag{7}$$

$\forall m = 1, 2, \cdots, M$. Here, In the above, I_r, I_g, I_b, $L_{\min}$, $L_{\max}$, η_r, η_g, η_b, are all prior known and preset positive scalars. Oftentimes, $I_\mathrm{r} = I_\mathrm{g} = I_\mathrm{b} = L_{\max}$ and $L_{\min} \approx L_{\max}$.

Each constellation symbol $\mathbf{s}_m$ must also satisfy the following "color constraint":

$$\sum_{m=1}^{M} \mathbf{s}_i = LM\mathbf{d}, \tag{8}$$

with L and the vector $\mathbf{d}$ preset.

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[1]This section is based on [1]–[3]. Please refer to them for details.

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II. The Optimization Strategy

To numerically solve the abovementioned constrained optimization, the MATLAB built-in subroutine of "fmincon" will be used. This "fmincon" may be set by the human user to realize an "interior point method" (also called a "barrier method") or an "active set" method. Both methods require the optimization problem to be convex, which is the optimization problem in Section I is not.

The "soft maximum" approximation is used in [1]–[3] to render Section I's optimization problem to become convex. The "soft maximum" approximation identifies the maximum as $\log \sum_{\forall j \neq k} e^{d_{j,k}}$, instead of $\max \{d_{j,k}, \forall j \neq k\}$.

The stopping criteria of "fmincon" are subsequently set to

(i) "TolCon" – the allowed constraint violation – is set to 10^{-12}.

(ii) "TolX" – the allowed variation in the output value – is set to 10^{-15}.

(iii) "TolFun" – the allowed constraint violation – is set to 10^{-15}.

Both "interior point method" (IPM) and the "active set" method (ASM) are started off with a random estimate generated by the MATLAB command "randn". As both methods are sensitive to the preset initial estimate, each method is run 30 times. Each $d_{\min}$ value presented subsequently is the best obtained in all 30 runs.

III. White CSK Constellation for a No-Cross-Talk Channel

The constellations below are designed for $\mathbf{H} = \mathbf{I}$ (i.e. a channel with no coupling across the three LED-channels) and constrained to white light, i.e. $\mathbf{d} = \frac{1}{3}[1, 1, 1]^T$, where ^{T} denotes transposition. Moreover, $L = 1.064, L_{\min} = 0.9$ and $I_{\mathrm{r}} = I_{\mathrm{g}} = I_{\mathrm{b}} = L_{\max} = 1.1$.

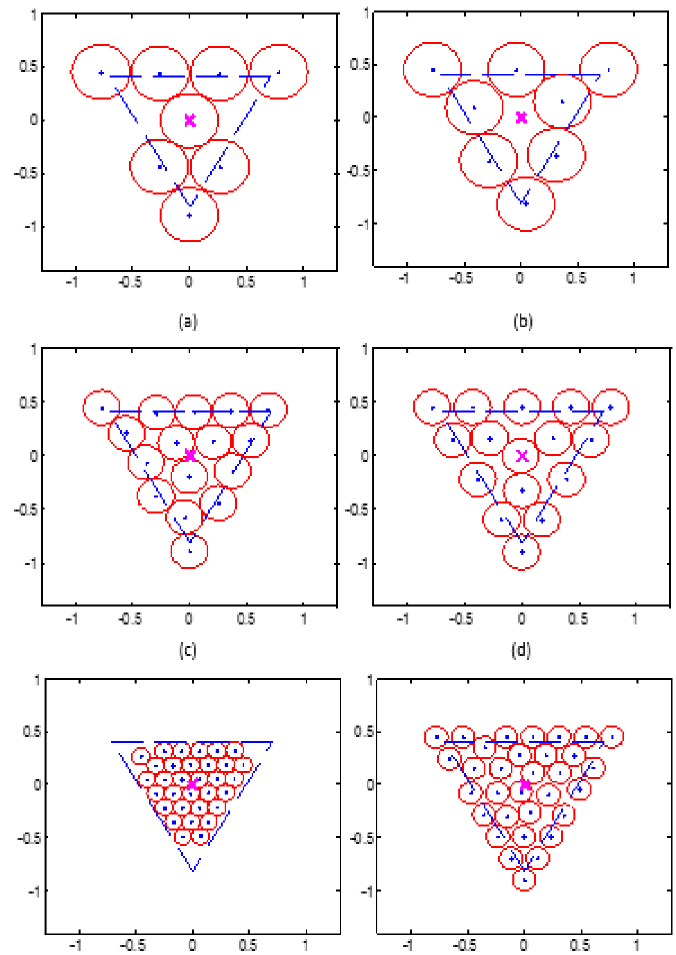

Fig. 1: Optimized constellation for
(a) $d_{\min} = 0.5236$ for $M = 8$ by IPM,
(b) $d_{\min} = 0.5362$ for $M = 8$ by ASM,
(c) $d_{\min} = 0.3190$ for $M = 16$ by IPM,
(d) $d_{\min} = 0.3194$ for $M = 16$ by ASM,
(e) $d_{\min} = 0.1532$ for $M = 32$ by IPM,
(f) $d_{\min} = 0.2036$ for $M = 32$ by ASM.

It may be observed that

(i) The "active set" method consistently gives a larger $d_{\min}$ than the "interior point method"

(ii) $d_{\min}$ decreases with an increasing M, as expected from the well known trade-off between bit-error rate and data rate.

IV. Red CSK Constellation for a Cross-Talk Channel

The constellations below are designed for $\mathbf{H} \neq \mathbf{I}$ (i.e. a channel with coupling across the three LED-channels) and constrained to red light with $\mathbf{d} = [0.3443, 0.4857, 0.1700]^T$. The values for $L, L_{\min}, L_{\max}, I_{\mathrm{r}}, I_{\mathrm{g}}$ and I_{b} are the same as in Section III.

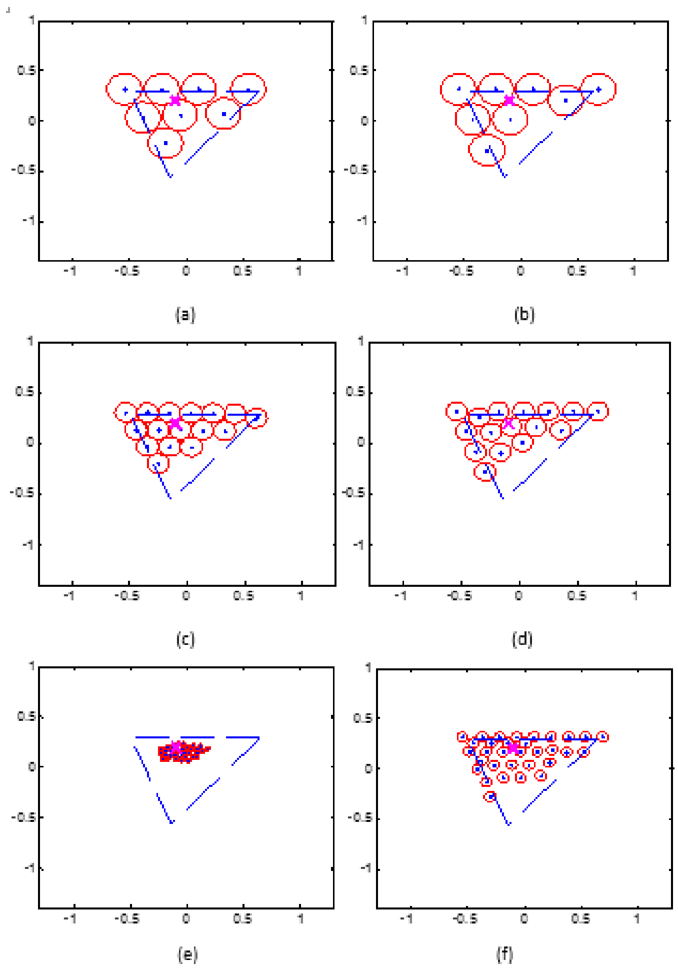

Fig. 2: Optimized constellation for
(a) $d_{\min} = 0.3072$ for $M = 8$ by IPM,
(b) $d_{\min} = 0.3067$ for $M = 8$ by ASM,
(c) $d_{\min} = 0.1895$ for $M = 16$ by IPM,
(d) $d_{\min} = 0.1805$ for $M = 16$ by ASM,
(e) $d_{\min} = 0.0422$ for $M = 32$ by IPM,
(f) $d_{\min} = 0.0959$ for $M = 32$ by ASM.

It may be observed that

(i) For $M = 32$ only, the "active set" method consistently gives a significantly larger $d_{\min}$ than the "interior point method". For $M = 8$ and $M = 16$, the "active set" method consistently gives a slightly smaller $d_{\min}$ than the "interior point method".

(ii) The $d_{\min}$ is significant lower in this case of $\mathbf{H} \neq \mathbf{I}$ and non-white light, than for Section III's case of $\mathbf{H} = \mathbf{I}$ and white light.

(iii) $d_{\min}$ decreases with an increasing M, as expected from the well known trade-off between bit-error rate and data rate.

References

[1] E. Monteiro and S. Hranilovic, "Constellation Design for Color-Shift Keying Using Interior Point Methods," *IEEE Workshop on Optical Wireless Communications*, 2012

[2] E. Monteiro, *Design and implementation of Color Shift Keying for Visible Light Communication*, M.Sc. thesis, Department of Electrical & Computer Engineering, McMaster University, September 2013.

[3] E. Monteiro and S. Hranilovic, "Design and Implementation of Color-Shift Keying for Visible Light Communications," *IEEE Journal of Lightwave Technology*, vol. 32, no. 10, pp. 2053-2060, May 2014.